NEWTON'S LAW OF COOLING

Newton's law of cooling *state that the rate of cooling (or rate of loss of heat) of a body is directly proportional to the temperature difference between the body and its surroundings, provided the temperature difference is small.*

Mathematical expressions for Newton's law of cooling. Consider a hot body at temperature T. Let T_0 be the temperature of its surroundings. According to Newton's law of cooling,

Rate of loss of heat $\propto$ Temperature difference between the body and its surroundings

or $$-\frac{dQ}{dt} \propto (T - T_0) \quad \text{or} \quad -\frac{dQ}{dt} = k(T - T_0) \qquad ...(1)$$

where k is a proportionality constant depending upon the area and nature of the surface of the body.

Let m be the mass and c the specific heat of the body at temperature T. If the temperature of the body falls by small amount dT in time dt, then the amount of heat lost is

$$dQ = mc\,dT$$

$\therefore$ Rate of loss of heat is given by $\frac{dQ}{dt} = mc\frac{dT}{dt}$

Combining the above equations, we get $-mc\frac{dT}{dt} = k(T - T_0)$

or $$\frac{dT}{dt} = -\frac{k}{mc}(T - T_0) = -K(T - T_0) \qquad ...(2)$$

where $K = k/mc$ is another constant. The negative sign indicates that as the time passes, the temperature of the body decreases.

The above equation can be written as $\frac{dT}{T - T_0} = -K\,dt$

On integrating both sides, we get $\int \frac{1}{T - T_0} dT = -K \int dt$

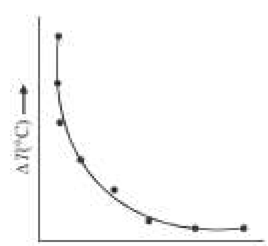
or $\log_e (T - T_0) = -Kt + c \qquad ...(3)$

or $T - T_0 = e^{-Kt + c}$

or $T = T_0 + e^c e^{-Kt}$

or $T = T_0 + C e^{-Kt} \qquad ...(4)$

where c is a constant of integration and $C = e^c$. Equations (1), (2), (3) and (4) are the different mathematical representations for Newton's law of cooling. Using equation (4), one can calculate the time of cooling of a body through a particular range of temperature. If we plot a graph by taking different values of temperature difference $\Delta T = T - T_0$ along y-axis and the corresponding values of t along x-along, we get a curve of the form shown in Fig. It clearly shows that the rate of cooling is higher initially and then decreases as the temperature of the body falls.



Experimental verification of Newton's law of cooling. The experimental set-up used for verifying Newton's law of cooling is shown in Fig. The set-up consists of a double walled vessel (V) containing water in between the two walls. A copper calorimeter (C) containing hot water is placed inside the double walled vessel. Two thermometers through the corks are used to note the temperatures T of hot water in calorimeter and T_0 of water in between the double walls respectively.